POLICY RESEARCH JOURNAL
ISSN (E): 3006-7030 (P) 3006-7022

Volume 3, Issue 1, 2025

Received: 01 December 2024
Accepted: 01 January 2025
Published: 08 January 2025

FIXED POINT THEOREM FOR MAPPING CONVEX CONTRACTING PERIMETER OF TRIANGLE AND APPLICATIONS

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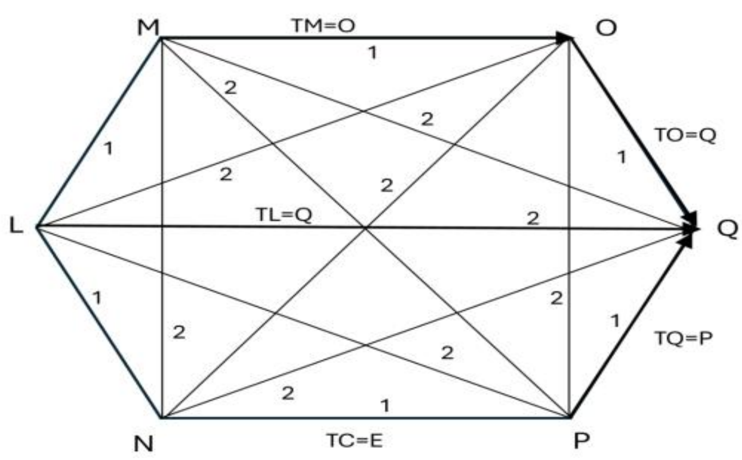
ABSTRACT

The main aim of this study is to analyze the generalized mapping contraction perimeter of a triangle in metric spaces. We also described a new approach to constructing proofs of some fixed-point theorems in metric spaces, and new tool for proving these theorems. The findings of this thesis consist of two major categories as listed below. In the first section we defined a new mapping known as convex contraction perimeter of a triangle and with its help established few fixed-point theorems in metric spaces. Thus, based on our results, we offered certain fixed - point results in metric spaces and the spaces which are equipped by binary relations. Moreover, we obtained several fixed points conclusions for these contraction mappings with the completion of metric spaces. In the second part of this paper, we used our results to show that a given system of integral equations has a common solution. We also integrated our findings to offer fix-point theorems employing specified mapping. Therefore, our findings add value to the already existing conclusions on the topics of interest within the literature. Finally, we proved the existence of fixed-point results in metric spaces associated with binary relations and have established our main results accompanied by examples.

Keywords: *Metric Space, Fixed Point Theorem, Contraction Mapping, Completeness, Generalized Contraction Mappings, Integral Equations.*

GRAPHICAL ABSTRACT



INTRODUCTION

In this research study, mappings in geometric spaces and relations between them with other portions of mathematics and science will be explored, in line with Singh & Burry (2020). Writing the given lesson, I realized that triangles are the most suitable objects for mathematical learning because these shapes are the most basic ones. Exploring mappings that shrink along the sides of triangles extends our understanding of shape deformations. There is no dearth of studies on fixed point theorems in general, but the current study looks at contractions from a triangle's boundary in particular. In this particular geometric setting, this work offers new findings about fixed points. Fixed point theory, which is generally theoretical, is used for computer science, engineering, physics, as well as economics. Studying mappings that contract along of a triangle's boundary the researcher can get a number of fruitful results useful for optimization, image processing, computational geometry, providing more effective algorithms and methods. The paper combines geometric and analytical methods and concepts with conceptions of dynamical systems, thereby enriching the mathematics and inspiring multi-disciplinary work. Studying these mappings generates important theoretical issues, including questions of existence and uniqueness and issues related to the analysis of iterative processes. Answering these questions further Pedagogy economics education helps us to deepen our knowledge of arithmetic. Hence, the study of contractive conditions along the boundary of a triangle is important from a theoretical and applied point in view, leading to future explorations of mathematical research. His work in June 2023 involved using Banach contraction principle on a triangle boundary. Banach principle has also stated that whenever a given metric has a complete space then all such contraction mapping must have a unique fixed point. In Petrov's conception, this principle is explored in a triangular scenario, with studies being made of the existence and non-uniqueness of fixed points and the performance of iterative processes in this situation (Petrov & Bisht, 2023). Later in December 2023, Petrov and Bisht generalized Kantorovic-type mappings of the perimeter of triangle and introduced generalized Kannan-mappings. The Kannan mappings are, in fact, a wider version of the contraction mappings and therefore have fixed points even under conditions of this nature. They successfully transplanted Kannan's ideas onto the boundary of the triangle and provided the existence of fixed points while studying the stability and convergence of such mappings. This research is useful when contraction conditions are relaxed and not restricted to the Banach space as we have seen in Bisht and Rakocevic (2020) and Petrov and Bisht (2023). C. M. Păcurar and O. Popescu studied generalized Chatterjea-type mappings on the boundary of a triangle on February 11, 2024. Chatterjea mappings, in which averages are used instead of the fixed contraction ratios, are designated for mappings with changing contractiveness. Their research is devoted to the presence and the exclusiveness of fixed points such that standard contraction metrics cannot be uniformly applied. This work extends knowledge of the Chatterjea mappings within triangles to the further reading literature in connection with PET (Petrov et al., 2024). These investigations enrich the understanding of the static relationship of the point-set in regards to geometric containment, which might open the new pleasant roads in the field of geometry, computation, and algorithm & design paradigms. It remains basic, due to contributions of Fréchet and Banach when the fixed-point theory remains crucial in nonlinear and functional analysis. Fixed point theory has contribution on the growth of the applied mathematics by presenting techniques to study questions in nonlinear analysis, optimisation and dynamical systems based on the notion of a metric space (Khan et al.,2024). Even now the applications of the fixed-point theory are still in the process of being discovered at the present time, as more areas in which the theory can be applied both theoretically conceptually as well as practically, are being explored.

CONVEX CONTRACTION MAPPING

A convex contraction mapping typically refers to a mapping $T : X \to \mathrm{X}$, where X is a convex subset

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ISSN (E): 3006-7030 (P) 3006-7022

of a metric or normed space, and T satisfies the following conditions:
T is a contraction mapping (there exists a, b∈ (0,1) such that $\forall\, x, y \in X$

$$d\left(T^2(x), T^2(y)\right) \leq a.\, d\left(T(x), T(Y)\right) + b.\, d(y, x).$$

$$where\ a + b < 1.$$

It is obliviously that this class of mapping contain class of contraction mappings. concerning the fix point of mapping which are convex contraction of order 2.

BANACH CONTRACTION MAPPING

Let (X, d) a metric space. A mapping $T: X \rightarrow X$ satisfies Banach contraction condition if there exists a sequence $k \in [0, 1)$ such that $\forall\, x, y \in X$, then inequality hold: T has a unique Fixed point $x \star \in X$:

$$d(T(x), T(y)) \leq K.\, d(y, x).$$

It is called Banach contraction mapping

KANNAN CONTARACTION MAPPING:

Let (X, d) a metric space. A mapping $T: X \rightarrow X$ satisfies Kannan's contraction condition if there exists a sequence $k \in [0, \frac{1}{2})$ such that $\forall\, x, y \in X$, then inequality hold:

$$d(T(x), T(y)) \leq k.[d(x, T(x) + d(y, T(y)].$$

It is called Kannan's contraction mapping.

CHATTERJEA'S CONTERACTION MAPPING:

Let (X, d) a metric space. A mapping $T: X \rightarrow X$ satisfies chatterjea's contraction condition if there exists a sequence $k \in [0, \frac{1}{2})$ such that $\forall\, x, y \in X$, then inequality hold:

$$d(Tx, Ty) \leq k.[d(x, Ty) + d(y, Tx)].$$

∀ x, y ∈ X, where k ∈ [0, 1/2).

REICH'S CONTRACTION MAPPING:

Let (X, d) be a metric space and $T: X \rightarrow X$ be mapping there exists the real numbers a, b and c satisfying: $a + b + c < 1, \forall\, x, y \in X$

$$\mathrm{d(Tx, Ty)} \leq \mathrm{a}.\, d(y, x) + b.\, d(x, Tx) + c.\, d(y, Ty).$$

Then, T is called Reich's contraction mapping.

GENRALIZED CONTRACTION MAPPING

Let (X, d) be a complete metric space with $|X| \geq 3$. We shall say that $T: X \rightarrow X$ is a mapping contracting perimeters of triangles on X if there exists $\alpha \in [0, 1)$ such that the inequality:

$$d(Tx, Ty) + d(Ty, Tz) + d(Tx, Tz) \leq \alpha.(d(x, y) + d(y, z) + d(x, z))$$

holds for all three pairwise distinct points x, y, $z \in X$. Also, this definition is equivalent to the definition of contraction mapping.

GENERALIZED KANNAN TYPE MAPPING:

Let (X, d) be a metric space with $|X| > 3$. We shall say that $T: X \rightarrow X$ is a generalized Kannan type mapping on X if there exists $\lambda \in [0, \frac{2}{3})$ such that the inequality:

$$d(Tx, Ty) + d(Tx, Ty) + d(Tx, Ty) \leq \lambda.(d(x, Tx) + d(y, Ty) + d(z, Tz))$$

holds for all three pairwise distinct points x, y, z ∈ X

GENERALIZED CHATTERJEA TYPE MAPPING

Let (X, d) be a metric space with $|X| \geq 3$. We shall say that $T: X \rightarrow X$ is a generalized Chatterjea type mapping on X if there exists $\lambda \in [0, \frac{1}{2})$, such that the inequality:

$$d(Tx, Ty) + d(Ty, Tz) + d(Tz, Tx) \leq \lambda.[d(x, Ty) + d(y, Tx) + d(y, Tz) + d(z, Tx) + d(z, Ty) + d(x, Tz)],$$

Holds for all three pairwise distinct points x, y, $z \in X$.

Generalized convex type mapping contracting perimeter of triangle:

This research explores the definition and characterization of the generalized mapping contraction perimeter of a triangle by extending the concept of generalized mapping contraction, the study develops the convex contraction perimeter of a triangle, enabling the derivation of new fixed-point theorems. These theorems are applied to systems of integral equations, demonstrating their relevance in proving the

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ISSN (E): 3006-7030 (P) 3006-7022

existence of common solutions and addressing key challenges in mathematical analysis.

Definition:

Let (X, d) be a metric space with $|X| \geq 3$. We shall say that $T: X \to X$ is a generalized convex type mapping contracting perimeters of triangles on X if there exists α, $\beta \in [0,1)$ such that the inequality

$$d(T^2 x, T^2 y) + d(T^2 y, T^2 z) + d(T^2 z, T^2 x) \leq \alpha.[d(Tx,Ty) + d(Ty,Tz) + d(Ty,Tx)] + \beta.[d(x,y) + d(y,z) + d(x,z)] \quad \ldots\ldots (1)$$

holds for all three pairwise distinct points x, y, $z \in X$.

Theorem: (New Convex Contracting Perimeters of Triangles)

Let (X, d) be a complete metric space with $|X| \geq 3$. We shall say that $T: X \to X$ is a convex contraction mapping on X i.e T is continuous if there exists α, $\beta \in [0,1)$ where $\alpha + \beta < 1$ s.t the inequality

$$d(T^2 x, T^2 y) + d(T^2 y, T^2 z) + d(T^2 x, T^2 z) \leq \alpha.[d(Tx,Ty) + d(Ty,Tz) + d(Ty,Tx)] + \beta.[d(x,y) + d(y,z) + d(x,z)]$$

holds for all three pairwise distinct points x, y, $z \in X$.

Then T has a unique fixed point more over for each $x \in X$ the picard iteration $\{xn\}$, which is define by:

$xn = Tx_{n-1}$ $\forall\, n \in N$ converge to fixed point of T.

Proof:

Let $x_0 \in X$ define picard itteration $\{xn\}$ in X, by $xn = Tx_{n-1}$ $\forall\, n \in N$ define by:

$$\delta = d(x_0, x_1) + d(x_1, x_2) + \ldots. + d(x_{n-1}, x_n)$$

And $K = \alpha + \beta \in [0,1)$ where n=0, 1, 2, ...by putting in (1),

Then:
$$d(x_2, x_3) + d(x_3, x_4) + d(x_2, x_4) = d(T^2 x_0, T^2 x_1) + d(T^2 x_1, T^2 x_2) + d(T^2 x_0, T^2 x_2)$$
$$\leq \alpha.[d(Tx_0, Tx_1) + d(Tx_1, Tx_2) + d(Tx_0, Tx_2)] + \beta.[d(x_0, x_1) + d(x_1, x_2) + d(x_0, x_2)] = \alpha.[d(x_1, x_2) + d(x_2, x_3) + d(x_1, x_3)] + \beta.[d(x_0, x_1) + d(x_1, x_2) + d(x_0, x_2)]$$
$$\leq \alpha.[d(x_1, x_2) + d(x_2, x_3) + d(x_1, x_3) + d(x_0, x_1) + d(x_0, x_2)] + \beta.[d(x_0, x_1) + d(x_1, x_2) + d(x_0, x_2) d(x_2, x_3) + d(x_1, x_3)]$$
$$= (\alpha + \beta)[d(x_0, x_1) + d(x_1, x_2) + d(x_1, x_2) d(x_2, x_3) + d(x_1, x_3)]$$
$$= (\alpha + \beta)(\delta)$$
$$= K(\delta) \quad (2)$$

Now:

$$d(x_4, x_5) + d(x_5, x_6) + d(x_4, x_6) = d(T^2 x_2, T^2 x_3) + d(T^2 x_3, T^2 x_4) + d(T^2 x_2, T^2 x_4)$$
$$\leq \alpha.[d(Tx_2, Tx_3) + d(Tx_3, Tx_4) + d(Tx_2, Tx_4)] + \beta.[d(x_2, x_3) + d(x_3, x_4) + d(x_2, x_4)]$$
$$= \alpha.[d(x_3, x_4) + d(x_4, x_5) + d(x_3, x_5)] + \beta.[d(x_2, x_3) + d(x_3, x_4) + d(x_2, x_4)]$$
$$\leq \alpha.[d(x_3, x_4) + d(x_4, x_5) + d(x_3, x_5) + d(x_2, x_3) + d(x_2, x_4)] + \beta.[d(x_3, x_4) + d(x_4, x_5) + d(x_3, x_5) + d(x_2, x_3) + d(x_2, x_4)]. And:$$
$$= (\alpha + \beta)[d(x_3, x_4) + d(x_4, x_5) + d(x_3, x_5) + d(x_2, x_3) + d(x_2, x_4)]$$
$$= (\alpha + \beta)(\delta)$$
$$= K(\delta). \quad (3)$$

$$d(x_6, x_7) + d(x_7, x_8) + d(x_6, x_8) = d(T^2 x_4, T^2 x_5) + d(T^2 x_5, T^2 x_6) + d(T^2 x_4, T^2 x_6)$$
$$\leq \alpha.[d(Tx_4, Tx_5) + d(Tx_5, Tx_6) + d(Tx_4, Tx_6)] + \beta.[d(x_4, x_5) + d(x_5, x_6) + d(x_4, x_6)]$$
$$= \alpha.[d(x_5, x_6) + d(x_6, x_7) + d(x_5, x_7)] + \beta.[d(x_4, x_5) + d(x_5, x_6) + d(x_4, x_6)]. And:$$

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$$\leq \alpha.[d(x_5,x_6)+d(x_6,x_7)+d(x_5,x_7)+d(x_4,x_5)+d(x_4,x_6)] + \beta.[d(x_5,x_6)+d(x_6,x_7)+d(x_5,x_7)+d(x_4,x_5)+d(x_4,x_6)]$$

$$= (\alpha+\beta)[d(x_3,x_4)+d(x_5,x_6)+d(x_6,x_7)+d(x_5,x_7)+d(x_4,x_5)+d(x_4,x_6)]$$

$$= \alpha(K\delta)+\beta(K\delta)$$

$$= (\alpha+\beta)(K\delta)$$

$$= K(K\delta)$$

$$= (K^2\delta). \quad (4)$$

Now

$$d(x_8,x_9)+d(x_9,x_{10})+d(x_8,x_{10}) = d(T^2x_6,T^2x_7)+d(T^2x_7,T^2x_8)+d(T^2x_6,T^2x_8).$$

$$\leq \alpha.[d(Tx_6,Tx_7)+d(Tx_7,Tx_8)+d(Tx_6,Tx_8)] + \beta.[d(x_6,x_7)+d(x_7,x_8)+d(x_6,x_8)]$$

$$= \alpha.[d(x_7,x_8)+d(x_8,x_9)+d(x_7,x_9)] + \beta.[d(x_6,x_7)+d(x_7,x_8)+d(x_6,x_8)$$

$$\leq \alpha.[d(x_7,x_8)+d(x_8,x_9)+d(x_7,x_9)+d(x_6,x_7)+d(x_6,x_8)] + \beta.[d(x_7,x_8)+d(x_8,x_9)+d(x_7,x_9)+d(x_6,x7)+d(x_6,x_8)]$$

$$= (\alpha+\beta)[d(x_7,x_8)+d(x_8,x_9)+d(x_7,x_9)+d(x_6,x_7)+d(x_6,x_8)]$$

$$= \alpha(K\delta)+\beta(K\delta)$$

$$= (\alpha+\beta)(K\delta)$$

$$= K(K\delta)$$

$$= (K^2\delta) \quad (5)$$

By above relation we obtain:

$$d(x_n,x_{n+1}) = \begin{cases} K^{\frac{n-1}{2}} & if\ n\ is\ odd \\ K^{\frac{n}{2}} & if\ n\ is\ even \end{cases}$$

Now we will show that $\{xn\}$ is a cauchy sequence

Let , $n \in N$ such that $n < m$, we will divide it into 2 cases

Case 1

If n is odd we have

$$d(x_n,x_m) \leq d(x_n,x_{n+1})+d(x_{n+1},x_{n+2}) + d(x_{n+2},x_{n+3})+\ldots\ldots+d(x_{nm-1},x_m)$$

$$\leq K^{\frac{n-1}{2}}+K^{\frac{n}{2}}+K^{\frac{n+1}{2}}+K^{\frac{n+2}{2}}+\ldots.$$

$$\leq 2.\left[K^{\frac{n-1}{2}}+K^{\frac{n}{2}}+K^{\frac{n+1}{2}}+K^{\frac{n+2}{2}}+\ldots.\right]$$

$$= 2.\frac{K^{\frac{n-1}{2}}}{1-K}\delta$$

Case 2

If n is even, we have

$$d(x_n,x_m) \leq d(x_n,x_{n+1})+d(x_{n+1},x_{n+2}) + d(x_{n+2},x_{n+3})+\ldots\ldots+d(x_{m-1},x_m)$$

$$\leq K^{\frac{n}{2}}+K^{\frac{n+1}{2}}+K^{\frac{n+2}{2}}+K^{\frac{n+3}{2}}\ldots.$$

$$\leq 2.\left[K^{\frac{n}{2}}+K^{\frac{n+1}{2}}+K^{\frac{n+2}{2}}+K^{\frac{n+3}{2}}+\ldots.\right]$$

$$= 2.\frac{K^{\frac{n}{2}}}{1-K}\delta$$

For all cases, we get

$$d(x_n,x_{n+1}) = \begin{cases} 2.\left(\frac{K^{\frac{n-1}{2}}}{1-K}\delta\right) & if\ n\ is\ odd \\ 2.\left(\frac{K^{\frac{n}{2}}}{1-K}\delta\right) & if\ n\ is\ even \end{cases}$$

By taking the limit as n, $m \to \infty$ in the above relation, we get

$d\ (xn,\ oxm) \to 0$ and so $\{xn\}$ is a cauchy sequence.

By completenes of X, we obtain:

$xn \to z$ as $n \to \infty$ for some $x \in X$

Since T is continuous, we obtain:

$$T_z = T\left(\lim_{n\to\infty} X_n\right) = \lim_{n\to\infty}(TX_n) = \lim_{n\to\infty}(X_{n+1}) = z.$$

Then z is a fixed point of T finally, we will prove that T has a unique fixed point.

Suppose that u, v are fixed point other than fixed point of T then

$$d(z,u)+d(u,v)+d(z,v) = d(Tz,Tu)+d(Tu,Tv)+d(Tz,Tv)$$

$$= d(T^2z,T^2u)+d(T^2u,T^2v)+d(T^2z,T^2v)$$

$$\leq \alpha.[d(Tz,Tu)+d(Tu,Tv)+d(Tz,Tv)] + \beta.[d(z,u)+d(u,v)+d(z,v)]$$

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ISSN (E): 3006-7030 (P) 3006-7022

$$= \alpha.[d(z,u) + d(u,v) + d(z,v)] + \beta.[d(z,u) + d(u,v) + d(z,v)]$$
$$= (\alpha + \beta)[d(z,u) + d(u,v) + d(z,v)].$$

This implies that:
$d(z,u) = 0, d(u,v) = 0, d(z,v) = 0$
And

$$z = u, u = v, v = z \text{ or } z = u = v$$

Hence proved

Applications:
Fixed point theory plays important role in nonlinear analysis and in applications of mathematical analysis. Banach contraction mapping principle is named to Banach who initiated fixed point theory with existence and uniqueness issue for solutions. This principle plays a significant role in solving the equations inolving integrals, able to marry both the theoretical and the applied, used in the fields of pure and Applied Mathematics, Physics, Engineers e Computational Sciences: Fredholm Nonlinear Integral Equations as well as Nonlinear Differential Equations from the Context of Computational Mathematics.

$$\boldsymbol{x(t) = v(t) + \frac{1}{|b-a|}\left|\int_a^b K(t, s, x(s))ds\right|}$$

Addressing temperature distribution using Volterra equations.

$$\boldsymbol{|x(t) - y(t)| = \int_a^b L(t,s)|x(s) - y(s)|ds\ b\ a}\ .$$

Approximating the solutions using iterative methods based on the fixed-point theorems, Fractional Integral Equations contain non-integers derivatives or integrals, and encounter applications in viscoelasticity, diffusion processes and signal processing. In the same way, Fractal Integral Equations are developed using fractal geometry to extend the traditional equations to its domain.
The analogous of the Banach principle with respect to these environments guarantee solvibility of problems under appropriate contractive assumptions. Banach's contraction principle is extended beyond it in convex contractions whereby fixed points considered are now convex combinations of the original two. Such mappings are valuable for solving an integral equation in a situation when basic contraction mappings are inapplicable and are employed in the sphere of mathematical and applied sciences for the treatment of nonlinear and interacting systems of integral equations.

$$\boldsymbol{T(x) = v(t) + \int_0^1 K(t, s, x(s))ds}$$

The existence and uniqueness of solutions to nonlinear integral equations with a nonlinear kernel is guaranteed by convex contraction mappings. Generalized convex contractions extend these ideas and provide an approximate fixed-point theorem for these transformations. Mostly reformulated as integral equations, these problems take advantage of convex contraction techniques, especially when the integral operator does not admit strict contraction criteria. This section introduces generalized convex contractions in the most general terms and shows that approximate fixed point of such maps exists in any complete metric space, which is preceded by an example to support our assumption and to illustrate the relevance of the results. The results established in this paper generalize and provide unified versions of the pre-existing results on fixed-point theorems. Convex contracting parameters have been generalized from traditional contraction principles to solve integral equations and the Banach contraction principle can be applied in more flexible settings in nonlinear integral equations because standard contraction conditions are sometimes too strong.

$$d(T^2x, T^2y) + d(T^2y, T^2z) + d(T^2x, T^2\ ) \le \alpha.[d(Tx, Ty) + d(Ty, Tz) + d(Ty, Tx)] + \beta.[d(x,y) + d(y,z) + d(x,z)]$$

Approximations with nonlinear kernel can be obtained by convex contracting parameters, which solve integral equations that describe coupled physical phenomena in biology or physics, and involve interaction terms which are weighted by convex combinations.

CONCLUSION

In this paper we consider the properties of some generalized convex type mappings in metric spaces and focus on the contraction of triangle perimeters. We analyze these mappings and in various cases examine their fixed and periodic points. It is demonstrated that these maps are continuous (in fact if being applied to an isolated point, \ (T \) is continuous). The theorems fix a connection between fixed points and periodic points in complete metric spaces. If \ (T \) contracts all triangle perimeters a fixed point exists if and only if the latter is true for \ (T \). In fact, such points are limited to two, and are interesting. If \ (T\) does not have a periodic point of period 2, then any sequence generated by \ (T \) converges to a fixed point. In fact, the conditions on contracting perimeters that were assumed to guarantee uniqueness of a fixed point, conflict with the existence of a fixed point given the mapping. Examples illustrate the theoretical results: One demonstrates that for convex mappings with triangle perimeter contracting contraction, there are exactly two fixed points on the other hand, one proves a convex mapping without any fixed points but with periodic points of period 2 still. We provide the first theoretical framework for generalized convex type mappings which contract triangle perimeters for fixed point existence and continuity in response to our study. From a point of view, the results relate fixed points to periodic points and the contraction of triangle perimeters, making the behavior of these mappings in metric spaces complete.

ACKNOWLEDGEMENTS

We thank Dr. Sayed Irtaza Hassnain from Department of Mathematics, Times Institute Multan, for helpful suggestions and complete assistance with preparing figures and manuscript outline.

Author Contributions

M.K wrote the main manuscript Draft, writing and M. A. A. computational work, S. I. H supervision, studied Data validation and M.A.A Editing, Reviewing. All authors reviewed the manuscript outline.

Funding

No external fundings.

Data Availability

No datasets were generated or analysed during the current study.

Declarations

Competing Interests

There are no conflicts of interest exist among authors of this research paper.

Consent to participate

Not applicable.

Consent for publications

The article contains no such material that may be awful, defamatory, or which would, if published, in any way whatsoever, violate the terms and conditions as laid down in the agreement.

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